Zapotec: A Coupled Euler-Lagrange Program for Modeling Earth Penetration

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The development of Zapotec, a coupled Euler-Lagrange computer code, is described. Zapotec is well suited for modeling earth penetration applications, and its utility is demonstrated for two benchmarks. These benchmarks model impacts of ogive-nose steel penetrators into Antelope and Sidewinder tuff. Comparisons are drawn with both the measured depth of penetration and deceleration history.

Introduction

Earth penetration is difficult to model, largely due to the transient, coupled nature of the impact event. The interaction between the penetrator and target is inherently coupled due to the vastly differing material response. The loading induced on the penetrator can lead to significant deformation and potentially high shock levels on any on-board components. In turn, the penetrator deformation affects the stress field in the neighboring target material. Empirical and engineering models have been used to model earth penetration (e.g., see [1,2,3,4]); however, these models have severe limitations when the penetrator exhibits significant bending. Physics-based approaches, such as Lagrangian or Eulerian methods, can afford a better treatment of the problem, but still have some disadvantages. Lagrangian methods are limited by mesh distortion associated with large target material deformations, while Eulerian methods have difficulties tracking motion of material interfaces. A coupled Euler-Lagrange solution approach can overcome the weaknesses associated with the two methods, allowing for solution of a class of problems not readily solved by either method alone.

Herein we describe the development of Zapotec, a coupled Euler-Lagrange computer code [5,6]. Zapotec is well suited for modeling earth penetration applications. For these applications, the earthen target is best modeled using an Eulerian solution approach due to the large material deformations involved. Conversely, the penetrator is best modeled using a Lagrangian approach as structural response is of primary interest and there is limited material deformation. By utilizing the strengths of the two solution methods, Zapotec can provide a more robust approach for modeling the problem.

In this paper, we first describe the Zapotec methodology. Then we demonstrate the utility of the coupling algorithm for two benchmark problems. These benchmarks model impacts of ogive-nose steel penetrators into Antelope and Sidewinder tuff. Comparisons are drawn with both the measured depth of penetration and deceleration history.

Zapotec Methodology

The coupled Euler-Lagrange solution approach utilized by Zapotec can be described as a framework that tightly couples two analysis codes, CTH and Pronto3D. CTH performs the Eulerian portion of the analysis, while Pronto3D performs the Lagrangian calculations. The two codes are run concurrently with the appropriate portions of a problem solved on their respective computational domains. All three codes support 3D problem development on massively parallel architectures. A brief description of the two codes and the Zapotec coupling algorithm follows.

CTH is an explicit, Eulerian shock physics code [7,8]. For a given time step, CTH utilizes a two-step approach for the solution of the conservation equations. The two-step solution approach first involves a Lagrangian step, where the Eulerian mesh is allowed to deform. This deformation provides an indication of material motion through the fixed, CTH reference mesh. The Lagrangian step is followed by a remap step. The remap algorithm advects material quantities (i.e., the volume flux, mass, momentum, and energy) from the deformed Lagrangian configuration back into the fixed reference mesh. Material interfaces are not explicitly tracked within CTH. They are reconstructed using the Sandia Modified Young's Reconstruction Algorithm. With this algorithm, material interfaces are reconstructed based on the volume fraction of material in the cell of interest and its neighbors. Mixed material cells also require special treatment. The CTH user has two options for treating yield in mixed material cells, referred to as mix options 3 and 5. Mix option 3 computes the mixed cell yield strength based on a volume-averaged yield strength for materials that can support shear. This option creates a "sticky" interface between materials, with slip controlled by the volume-averaged yield strength of the materials residing in the mixed cell. Mix option 5 sets the mixed cell yield strength to zero, effectively creating a frictionless surface between materials.

Pronto3D is an explicit Lagrangian, finite element (FE) code developed for modeling transient solid mechanics problems involving large deformations and contact [9,10]. The numerical formulation uses an updated Lagrangian approach with the Cauchy stress as the stress measure. As is customary for explicit FE codes, under-integrated elements are used in the formulation to avoid potential mesh locking associated with material incompressibility and provide a computational savings compared with fully integrated elements. The under-integrated elements require hourglass control to eliminate energy-less modes of deformation. Constitutive behavior is evaluated with respect to the unrotated reference frame. This requires determination of the material rotation at each time step, but has the advantage of allowing material models to be cast without regard to finite rotations. A nodal constraint approach is taken for contact enforcement, with corrections made to the nodal accelerations to enforce the contact conditions.

Zapotec controls both the time synchronization between CTH and Pronto3D as well as the interaction between materials on their respective computational domains. The time synchronization is depicted in Figure 1. At a given time t_n, Zapotec performs the coupled treatment between the Eulerian and Lagrangian materials in the problem. Once this treatment is complete, both CTH and Pronto3D are run independently over the next Zapotec time step. In general, the Pronto3D stable time step will be smaller than that for CTH. When this occurs, Zapotec allows subcycling of the Pronto3D code for computational efficiency and accuracy. The Pronto3D subcycling is illustrated in Figure 1. Zapotec ensures the two codes reach the same time t_{n+1} at the end of the Zapotec time step.

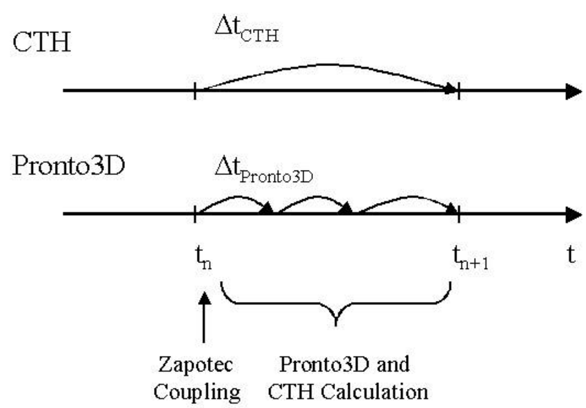

Figure 1. Time synchronization of CTH and Pronto3D

An outline of the coupling algorithm is provided in Figure 2. The coupled treatment at time t_n involves getting data from CTH and Pronto3D, working on that data, then passing the updated data back to the two codes. Zapotec first operates on the CTH data, a process termed material insertion. This involves mapping the current configuration (and state) of a Lagrangian body onto the fixed Eulerian mesh. The insertion algorithm determines what portions of a Lagrangian body are overlapping the CTH mesh. Material state data from the overlapping Lagrangian body are then mapped into cells in the CTH mesh. Mapped data includes the mass, momentum, stress, sound speed, and internal energy. In general, a CTH cell will be overlapped by several Lagrangian elements. When this occurs, the mapped Lagrangian quantities for each element are weighted by their volume overlap. The exception is the deviatoric stress, which is mass-weighted. The weighted quantities are accumulated for all elements overlapping a cell, after which the intrinsic value is recovered for insertion. The inserted data are then passed back to CTH as a mesh update.

Once the material insertion is complete, the external loading on the Lagrangian material surfaces is determined from the stress state in the Eulerian mesh. Since the Lagrangian material surface is uniquely defined, it is straightforward to determine the external forces on a Lagrangian surface element from the traction vector, the element surface normal, and element area. Zapotec has the capability to evaluate frictional contact, which is currently based on a

Coulomb friction model. The applied loads are passed back to Pronto3D as a set of external nodal forces. Once the coupled treatment is complete, both CTH and Pronto3D are run independently with their updated data.

(1) Remove pre-existing Lagrangian material from the CTH mesh
(2) Get updated Lagrangian data
(3) Insert Lagrangian material into the CTH mesh
  (a) Compute volume overlaps
  (b) Map Lagrangian data – mass, momentum, stress, sound speed, internal energy
  (c) Pass updated mesh data back to CTH
(4) Compute external forces on Lagrangian surface
  (a) Determine surface overlaps
  (b) Compute surface tractions based on Eulerian stress state
  (c) Compute normal force on element surface
  (d) If friction, compute tangential force
  (e) Distribute forces to nodes and pass data back to Pronto3D
(5) Execute Pronto3D and CTH

Figure 2. Summary of the Zapotec coupling algorithm

Antelope Tuff

Longcope and Forrestal [3] evaluated a cavity-expansion approach for modeling earth penetration. A test involving the normal impact of an ogive-nose steel penetrator into an Antelope tuff target at the Sandia Tonopah Test Range (TTR), Nevada served as a benchmark for validation of their cavity-expansion model. This test will also serve as a benchmark for Zapotec. In this test, a Davis Gun was used to launch the penetrator into the target with an impact velocity of 520 m/s. The length, aft-body diameter, and mass of the 6 caliber-radius-head ogive-nose penetrator were 1.56 m, 0.156 m, and 162 kg, respectively. A 55.3 kg pusher plate, fitting the bore of the gun, was attached to the aft end of the penetrator. The pusher plate provided an added mass contribution during the initial stages of penetration. An onboard accelerometer was used to measure the penetrator's deceleration history, which in turn, was integrated to obtain the depth of penetration. For post-test analysis, the deceleration data was filtered to 500 Hz to obtain the rigid body decelerations.

The Antelope tuff target is a partially welded ash flow tuff, which can be classified as a very low strength rock [11]. Field cores characterizing the tuff indicated the flow was over 7.5 m thick and was generally unfractured both horizontally and vertically. Isolated pockets of weaker material were noted at various locations in the flow. Laboratory testing of samples taken from the field cores was conducted to further characterize the material. The tuff exhibited considerable variability, which is evident in the triaxial test data shown in Figure 3. The tensile strength was estimated as 1.36 MPa. The sample depths for cores AT-8 and AT-9 are unknown.

For a Zapotec analysis, the user must develop the CTH and Pronto3D model inputs independently. Model inputs include development of the computational mesh, material specifications, and assignment of initial and boundary conditions. At the outset of a Zapotec analysis, the user categorizes materials as Lagrangian or Eulerian. Here the target is considered an Eulerian material, while the penetrator is Lagrangian. The target was modeled in CTH as a semi-infinite domain. The CTH mesh was uniformly meshed in the interaction region at a 1 cm cell resolution. The mesh was graded away from the interaction region with the mesh extents modeled as absorbing boundaries. For the Pronto3D analysis, a FE model was developed for both the penetrator and pusher plate (see Figure 4(a)). Lagrangian contact was specified between the two bodies, allowing them to separate after full embedment of the penetrator (see Figure 4(b)). Both the penetrator and pusher plate were assigned an initial velocity of 520 m/s, with the velocity vector oriented normal to the target. Frictional contact was assumed between the penetrator and target, with an assumed friction coefficient of 10 percent. The CTH mix option 5 was specified to allow Zapotec to control the frictional contact between the penetrator and target. Quarter symmetry was also assumed in the problem development to reduce the problem size.

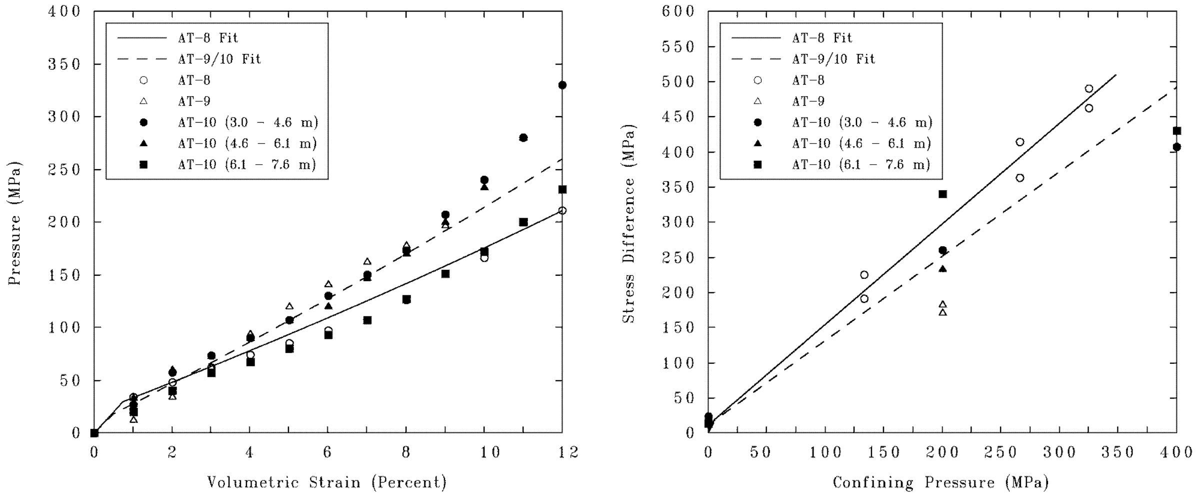

Figure 3. Antelope tuff material response from triaxial testing

(a) t = 0 msec (b) t = 4 msec

Figure 4. Calculated penetration into Antelope tuff

Prior to performing the Zapotec analysis, it was necessary to develop a constitutive relationship for the Antelope tuff. Since the target is considered an Eulerian material, its material behavior is described within CTH. CTH decouples the material behavior into its dilatational and deviatoric response. The dilatational behavior is described by an equation of state (EOS), while a strength model describes the deviatoric response. It was determined that the material response would be best replicated using an EOS with porosity (i.e., the P-α model) and the geologic strength model. The P-α EOS is commonly used to model porous materials, where α is a state variable that tracks the crushing of the pores until the fully dense material response is recovered. The geologic strength model allows a pressure-dependent yield surface. The CTH material model driver was used to determine the material parameters for both the EOS and strength model. Due to the scatter in the triaxial data, two separate data fits were made to bracket the material response. The resulting fits are shown in Figure 3 and are denoted as the AT-8 and AT-9/10 fits based on the data set used. The CTH material parameters associated with these fits are provided in Tables 1 and 2. In Figure 3, it is apparent the hydrostatic response at large volumetric strains is significantly under-predicted with the P-α model. This could possibly lead to an under-prediction of the penetrator loading during the initial stages of penetration, where impact pressures are relatively large.

The material modeling for the penetrator and pusher plate was straightforward. The steel for the two bodies was modeled as an elastic-plastic hydrodynamic material within Pronto3D. The dilatational response was modeled using a Mie-Gruneisen EOS (ρ_o = 7883 kg/m^3, c = 4692 m/s, s = 1.73, Γ = 1.67). The deviatoric response is modeled using a bilinear elastic-plastic material response assuming isotropic hardening (E = 206.9 GPa, ν = 0.3, σ_o = 1.23 GPa, H = 713 MPa).

Table 1. CTH material parameters for P-α EOS

Material Fit	ρ_p (kg/m^3)	ρ_m (kg/m^3)	c_s (m/s)	s	p_s (GPa)	p_e (MPa)	c_e (m/s)	Porosity (%)
AT-8	1720	2550	4000	1.54	1.0	20	1500	33
AT-10	1610	2550	3000	1.54	1.6	10	1500	33
Sidewinder	1800	2500	2650	1.54	2.5	330	2000	28

ρ_p – Initial porous material density
ρ_m – Matrix material density
c_s – Sound speed
s – Linear coefficient of us-up curve
p_s – Compaction pressure
p_e – Elastic pressure
c_e – Sound speed in elastic pore compaction region

Table 2. CTH material parameters for geologic strength model

Material Fit	ν	σ_o (MPa)	σ_{max} (GPa)	dy/dp
AT-8	0.26	11.47	1.0	-1.43
AT-10	0.22	11.47	0.5	-1.20
Sidewinder	0.21	50.0	0.6	-1.30

ν - Poissons ratio
σ_o – Yield strength at zero pressure
σ_{max} – Yield strength
dy/dp – Slope of the yield surface at zero pressure

Comparisons of the measured and calculated deceleration histories are provided in Figure 5. The calculated decelerations were filtered to 500 kHz for direct comparison with the test data. The measured deceleration history is characterized by a region of high axial decelerations, which is then followed by a region of fairly constant deceleration. Late in time, as the penetrator comes to rest, there is a jump in the measured decelerations. The magnitude and timing of the peak decelerations compare well between calculation and test (approximately 2800 g for the test compared with 2375 and 2770 g for the AT-8 and AT-9/10 fits, respectively). However, significant differences are noted later in time, where both calculations severely under predict the decelerations. The decelerations were integrated to obtain the depth of penetration. The measured depth of penetration was 7.9 m. The calculated depths of penetration for the AT-8 and AT-9/10 fits were in excess of 10 m (the depth could not be determined since the penetrator ran off the mesh). This is consistent with the severe under-prediction of the decelerations later in time.

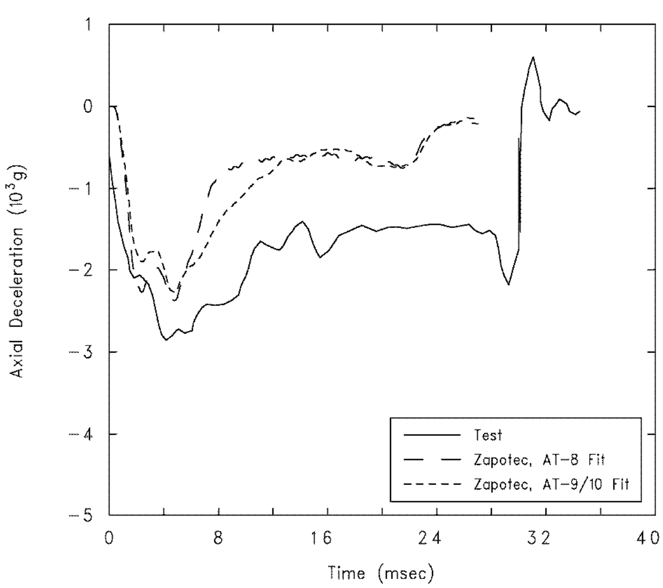

Figure 5. Deceleration history for Antelope tuff target

The reasons for the differences between calculation and test are unclear. Uncertainties associated with modeling the constitutive behavior of the Antelope tuff certainly contribute to the differences. Not only was there significant scatter in the triaxial data, but also the material fits of this data were problematic. In particular, the CTH P-alpha model significantly under-predicted the hydrostat at large volumetric strains. Also, the measured yield strength varied significantly with depth (see data for AT-10). In the calculation, the target was modeled as a homogenous body and no attempt was made to model the response as a function of depth. The choice of CTH fracture stress can also strongly influence the deceleration. The fracture model used by CTH is a PMIN-based model. With this model, the material pressure is relaxed to a state prior to fracture whenever the prescribed fracture criterion is exceeded. This is done with an iterative algorithm that increases the material density until a consistent relaxed pressure stat is achieved, while holding the energy constant. Void is then inserted into the cell to account for the change in material volume resulting from increasing the density. The void insertion is evident in the calculation (e.g., see Figure 4 (b)). In general, higher fracture stresses will improve the integrity of the material, which in turn, can lead to higher penetrator decelerations. No attempt was made to examine the influence of fracture stress on the calculation.

Uncertainties associated with modeling friction are also expected to contribute to differences between calculation and test. In the calculation, a constant Coulomb friction coefficient was assumed. A velocity-dependent friction model is likely more appropriate, particularly in the low velocity regime where there is expected to be increased resistance to penetration. Data for steel on steel contact indicates the friction coefficient is non-linear, with values ranging from 5 to 18 percent at sliding velocities of 600 and 20 m/s, respectively [12]. One could expect a greater variation for steel on tuff, particularly at the lower sliding velocities. The 10 percent constant coefficient of friction assumed for this analysis was thought to be an average value over the velocity regime of interest.

Sidewinder Tuff

Longcope [4] also modeled a Davis Gun test involving the impact of an ogive-nose steel penetrator into Sidewinder tuff at the TTR. This test will also serve as a benchmark for Zapotec. In this test, the penetrator impacted the target with a velocity of 463 m/s. The impact angle, as measured from the target surface, was 80 degrees. The length and outside diameter of the 3 caliber-radius-head ogive-nose penetrator were 1.43 and 0.272 m, respectively. The aft portion of the penetrator had a conical flare (see Figure 6). The flare length and maximum outside diameter were 0.338 and 0.305 m, respectively. Unlike the Antelope tuff test, the pusher plate was integrated with the penetrator. The total mass of the penetrator and integrated pusher plate was 411 kg. An onboard accelerometer was used to measure the penetrator's deceleration history, which in turn, was integrated to obtain the depth of penetration. For post-test analysis, the deceleration data was filtered to 500 Hz to obtain the rigid body decelerations.

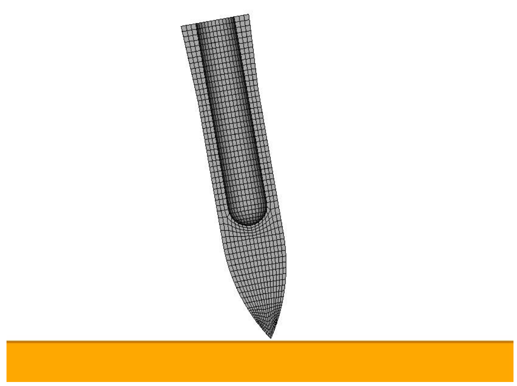

Figure 6. Penetrator configuration for impact into Sidewinder tuff

Sidewinder tuff is slightly harder than Antelope tuff and can be classified as a low to medium strength rock [13]. Field cores characterizing the tuff indicated the flow was approximately 3.0 m thick. The flow was generally unfractured along the horizontal, but exhibited some widely spaced vertical cracks. Laboratory testing of samples taken from the field cores was conducted to further characterize the material. The tuff exhibited considerable variability, which is evident in the triaxial test data shown in Figure 7. The tensile strength was estimated as 3.3

MPa. There was no information regarding the sample depth. Unfortunately, the range of the hydrostatic data was limited to 400 MPa. The hydostatic data was augmented with data for Mt. Helen tuff, which is slightly stiffer than Sidewinder tuff [14]. The augmented data is shown in Figure 7. The CTH material parameters used to fit the triaxial test data are provided in Tables 1 and 2.

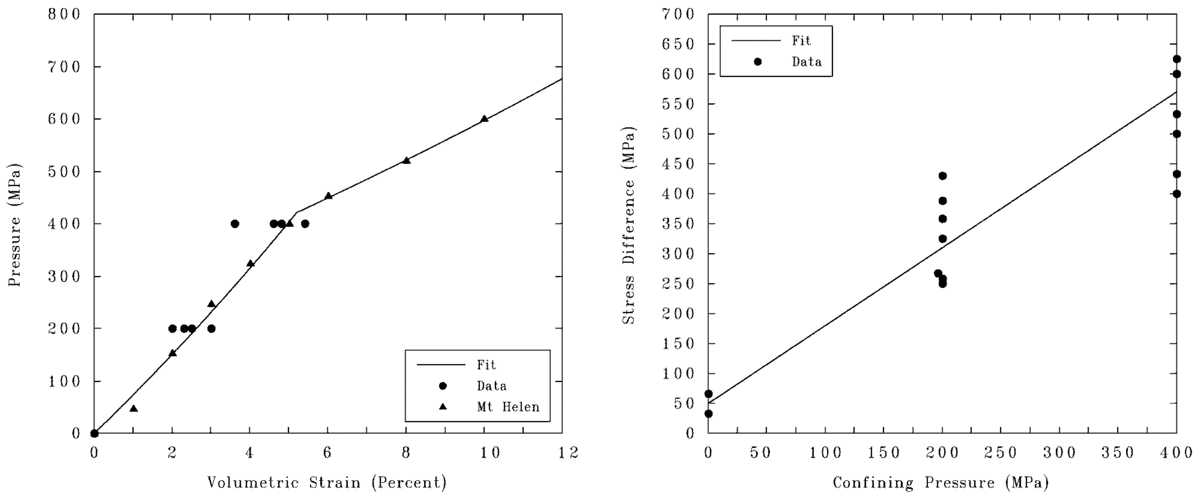

Figure 7. Sidewinder tuff material response from triaxial testing

The Zapotec problem development was much the same as discussed for the Antelope tuff test. Due to the oblique impact, half symmetry was assumed. The target was modeled in CTH as a semi-infinite domain. The CTH mesh was uniformly meshed in the interaction region at a 2 cm cell resolution. The mesh was graded away from the interaction region with the mesh extents modeled as absorbing boundaries. For the Pronto3D analysis, a FE model was developed for the penetrator that included the mass contribution of the integrated pusher plate. The penetrator was assigned an initial velocity of 463 m/s, with the velocity vector oriented 80 degrees from the target surface. Frictional contact was assumed between the penetrator and target, with an assumed friction coefficient of 10 percent. The CTH mix option 5 was specified to allow Zapotec to control friction between the penetrator and target.

Comparisons of the measured and calculated deceleration histories are provided in Figure 8. The calculated decelerations were filtered to 500 kHz for direct comparison with the test data. Both the magnitude of the peak deceleration and the general form of the deceleration history compare well; however, the calculation exhibits a greater rise time to the peak with the decelerations over-predicted following the initial peak. Once again the differences can likely be attributed to uncertainties in modeling both the material behavior and frictional effects. The measured depth of penetration was 2.9 m. The calculated depth was 2.4 m, which is consistent with the over-prediction in late time decelerations.

Concluding Remarks

Zapotec is a work in-progress, which shows great promise for modeling earth penetration. A major advantage of Zapotec over simpler solution approaches (e.g., empirical or engineering models) is that the actual problem can be modeled. This was evident for both benchmarks. In the case of the Antelope tuff test, both the penetrator and pusher plate were explicitly modeled and allowed to interact independently with the target (see Figure 4 (b)). In the case of the Sidewinder tuff test, the penetrator model included the added complexity of the aft flare. Not only can the actual geometry be modeled, but also the analyst has the flexibility to refine the finite element mesh in regions of interest independently of the Eulerian mesh. Although the target was modeled as a homogenous region, it would have been possible to model both horizontal layering and vertical fractures had more detailed site characterization data been available. Of course, the added flexibility to capture these features comes with a price. The calculation can be computationally intensive. Also, more detailed descriptions of the penetrator and target materials are required. This is especially difficult for geologic materials, where considerable variability is noted for the material properties. Zapotec can provide the flexibility to model more complex problems. This becomes important when penetrator structural response is an issue or when more complex targets are of interest (e.g., when considering layered targets or rock rubble).

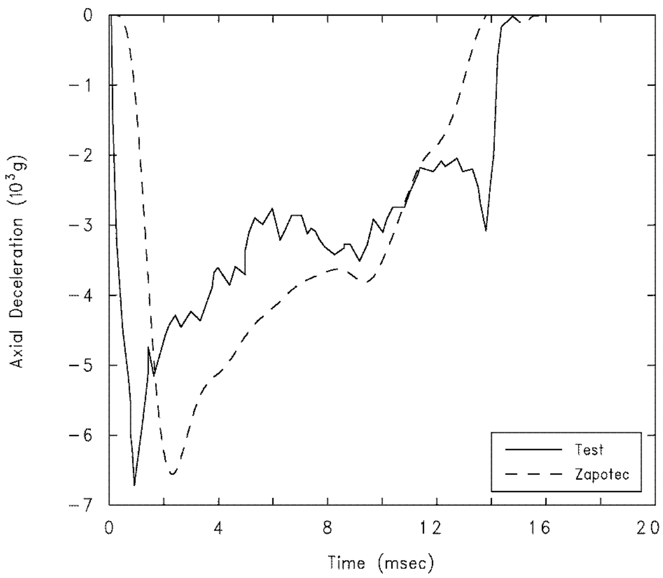

Figure 8. Deceleration history for Sidewinder tuff target

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